



# Quadratic Functions

Notes

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Keepin' it Real

Sandy works for a cruise line. The company prefers to set sail with as few empty rooms as possible. One ship has unoccupied rooms so Sandy is planning a group rate promotion to try to fill these rooms. Every additional room will be discounted depending on the amount of total rooms ( $x$ ) booked based on her quadratic model. When will each discount maximize profit? Help Sandy decide how much she should discount per each additional room to maximize the profit.



## Vocabulary

Write the definition of the term and include an image or example that represents it.

Term	Definition	Example
<i>Quadratic Function</i>		
<i>Quadratic Equation</i>		
<i>Quadratic Formula</i>		
<i>Roots</i>		
<i>Line of Symmetry</i>		
<i>Vertex Form</i>		
<i>Completing the Square</i>		

## Key Concept (a)

What is a Quadratic Function?	The Conceptualizer!
<p><i>Quadratic functions</i> are polynomial functions to the second degree. They can be written in the form <math>f(x) = ax^2 + bx + c</math>.</p> <p>When graphed they create parabolas that are noted for their “u-shape”. The base of the quadratic function is known as the <i>vertex</i> which will be the maximum or minimum of the function.</p>	

## Key Concept (b)

Solving Quadratic Equations by Factoring	The Conceptualizer!
<p>Knowing how <i>factoring</i> works help us solve quadratic equations.</p> <p>If we can factor the quadratic equation in the form of <math>(x + a)(x + b) = 0</math>, the equation is almost solved! This is because when</p> $(x + a)(x + b) = 0,$ $(x + a) = 0 \text{ or } (x + b) = 0.$ <p>Then we can solve the equation for the two cases separately. These equations reveal the roots of a quadratic function.</p>	

## Skill Practice #1

Solve Quadratic Equations	Notes
$x^2 - 2x - 15 = 0$	

## Key Concept (c)

Solving Quadratic Equations with Square Roots	Details
<p>In some cases, a quadratic expression will already be a perfect square on one side.</p> <p>If <math>x^2 = a</math>, then <math>x = \pm \sqrt{a}</math></p>	

## Skill Practice #2

Solving Quadratic Equations with Square Roots	Notes
$(x - 4)^2 = 10$	

## Key Concept (d)

Solving Quadratic Equations In Vertex Form	Details
<p>If the quadratic equation is in its <i>vertex form</i>, the square is already completed. We just need to do some additional algebra before we can solve with square roots.</p> <p>Here are the steps:</p> <ol style="list-style-type: none"> <li>1. Isolate the square.</li> <li>2. Take the square root on both sides (remember to use the sign <math>\pm</math> on the right hand side)</li> <li>3. Isolate x and simplify the solution. Sometimes it's more clear to leave it alone instead of getting decimals.</li> </ol>	

### Skill Practice #3

Solving Quadratic Equations In Vertex Form	Notes
$-4(x - 8)^2 - 3 = -11$	

### Key Concept (e)

Completing the Square	Details
<p>If an equation does not match the pattern of a perfect square or a factor by reverse FOIL, we can “complete the square”.</p> <p><i>Completing the square</i> makes the quadratic into a square by adding a proper constant based on the coefficient of the x-term (b) to make it match the form of a perfect square.</p> <p>The constant added to the quadratic is the square of half of the coefficient of the x-term, <math>(\frac{b}{2})^2</math>.</p> <p>CAUTION: This method would only work if the coefficient of <math>x^2</math> is 1. Otherwise, we need to change the coefficient to 1 before completing square. It will require division.</p> <p>NOTE: This is the same process used to convert a quadratic in standard form to a quadratic in vertex form.</p>	

## Skill Practice #4

Completing the Square	Notes
$3x^2 - 3x - 2 = 0$	

The steps to completing the square are always in the same order.

Step 1	
Step 2	
Step 3	
Step 4	
Step 5	
Step 6	

## Key Concept (f)

Quadratic Formula	Details
<p>If in doubt, we can solve any quadratic equation with the <i>quadratic formula</i>.</p> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ <p>where a, b and c are the coefficients and constant when the quadratic equation is in its standard form <math>ax^2 + bx + c = 0</math>.</p> <p>It works for any quadratic equation as long as it is in its standard form.</p> <p>It just may not be the fastest in all cases. #slowbutsteady</p> <p>#WatchYourNegativeSigns</p>	

## Skill Practice #5

Quadratic Formula	Notes
$3x^2 + 4x - 5 = 0$	

## #Observations

### Unreal

In cases, where quadratics have no solution (meaning they do not have any real roots), it is most clearly seen in the quadratic formula because  $b^2 - 4ac$  will be negative.

## Key Concept (g)

Graphs of Quadratic Equations	The Conceptualizer!
<p>Graphs for quadratic equations are <i>parabolas</i>.</p> <p>We can tell the general look of the parabola from the general form of the quadratic equation <math>y = ax^2 + bx + c</math>.</p> <p>When <math>a</math> is positive, the parabola opens upward. When <math>a</math> is negative, the parabola opens downward.</p> <p>When <math> a  &gt; 1</math>, the parabola will be “skinnier”.</p> <p>When <math> a  &lt; 1</math> (meaning coefficient of <math>x^2</math> is a fraction less than one), the parabola will be “fatter”.</p>	

## Sort

Write examples of quadratic functions that will open upward or downward.

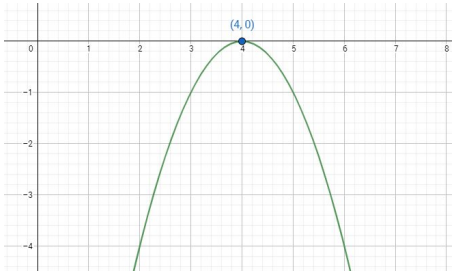
Open Up	Open Down

## Key Concept (h)

Maximum and Minimums of Quadratic Functions	The Conceptualizer!
<p>Parabolas are symmetrical. The vertex is the point where the parabola changes direction.</p> <p>The vertex is the maximum (for parabolas that open downward) and minimum (for parabola that open upward).</p> <p>We can identify the maximum and minimum two different ways:</p> <ul style="list-style-type: none"> <li>(1) when we are given a graph.</li> <li>(2) when we are given the equation in vertex form by identifying <math>(h,k)</math> and we consider the sign on the '<math>a</math>' coefficient.</li> </ul>	



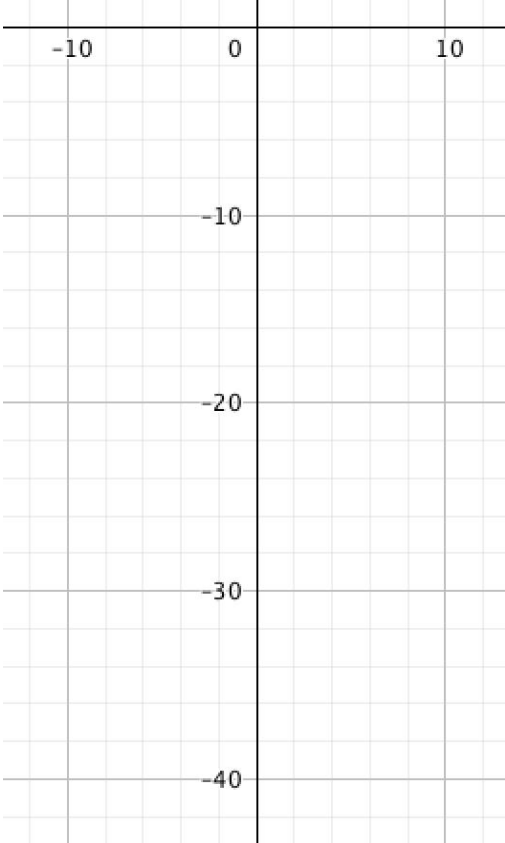
## Skill Practice #6

Maximum and Minimums of Quadratic Functions	Notes
	
$y = (x + 1)^2 + 3$	

## Key Concept (i)

Graphing Quadratic Equations	The Conceptualizer!
<p>Graphing quadratic equations is similar to graphing linear equations. The only difference is the graph is not a straight line but a smooth parabola.</p> <p>First, we need to evaluate quadratic functions at specific values of <math>x</math> you pick.</p> <p>It might be easier to start off with the <math>y</math>-intercept (when <math>x = 0</math>) and remember to pick values of <math>x</math> so you can sketch both sides of the parabola.</p> <p>The more points you pick, the more detailed your graph will be.</p> <p>Then sketch a smooth curve!</p> <p>Double Checks:</p> <ol style="list-style-type: none"> <li>(1) should your parabola open up or down?</li> <li>(2) parabola is symmetric</li> </ol>	

Skill Practice #7

Graphing Quadratic Equations	Notes
<p>Graphing <math>y = -x^2 + 2x - 5</math></p> 	

## Applications

What value of  $c$  in  $x^2 - 8x + c = 0$  gives the equation one real solution?

The product of two consecutive negative, odd integers is 1155. What is the larger integer?

While on vacation, Luke hikes up a mountain. He throws a rock at 17.2 meters per second from a 60 meter tall platform. The equation for the object's height  $h$  at time  $t$  seconds is  $h(t) = -4.9t^2 + 17.2t + 60$ . When will the rock hit the ground?